Math 4200 , Review 2

1. Definitions:

a) List the axioms for the real line.

b) State the Dedekind Principle.

c) What is a sequence?

d) Suppose $\{s_n\}$ is a sequence. Define what it means for $\{s_n\}$ to have limit s .

e) Suppose $\{s_n\}$ is a sequence. Define what it means for $\{s_n\}$ to have limit ∞ .

f) Suppose $\{s_n\}$ is a sequence. Define what it means for $\{s_n\}$ to be a Cauchy sequence.

g) Define what it means for the series $\sum_{i=1}^{\infty} a_i$ to converge.

2. Using the limit definition, prove that $\lim_{n\to\infty} \frac{2n}{3n-1} = \frac{2}{3}$.

3. Let S be a non empty subset of $\Re$ and suppose S is bounded above. Let m $= sup(S)$. Show that for each $\epsilon > 0$, there exists x in S such that $m - \epsilon < x \leq m$. (This is the "backaway principle" for suprema.)

4. Suppose the sequence $\{b_n\}$ is bounded. That is, there exists $M > 0$ such that $|b_n| \leq M$ for all n . If $\lim_{n\to\infty} a_n = 0$, show that $\lim_{n\to\infty} a_n b_n = 0$.

5. Suppose $\{s_n\}$ is a monotone, non increasing sequence; that is, for each n, $s_{n+1} \leq s_n$. Suppose also that $\{s_n\}$ is bounded below; that is, there exists a real number M such that for all n, $M \leq s_n$. Prove that $\{s_n\}$ converges.

6. State and prove the Squeeze Theorem for sequences.

7. Suppose the sequence $\{a_n\}$ converges to L . Show that $\{a_n\}$ is bounded, that is, $\exists\ M > 0$ such that $|a_n| < M \ \ \forall n \in J$.

8. Definition

Define what it means for a function f to have a limit L at $x = a$.
The <u>notation</u> is $\lim_{x \to a} f(x) = L$

9. State the Nested Intervals Theorem.

10. State the Bolzano-Weierstrass Theorem.

11. Suppose the sequence $\{a_n\}$ converges to L. Show that $\{a_n\}$ is a Cauchy sequence.

12. Suppose the sequence $\{a_n\}$ is a Cauchy sequence. Show that $\{a_n\}$ converges. *

13. Use the limit definition to show that $\lim_{x \to 4} \sqrt{x} = 2.$

14. Suppose f is defined on (∞, ∞). There are essentially three different ways for $f(x)$ to not have a limit at $x =$ a. State the three different ways and give a specific example of each.

15. Define what is meant by $\lim_{x \to (\ )} f(x) = (\ )$.

16. Suppose $\lim_{x \to a} f(x) = L$, $\lim_{x \to a} h(x) = L$, and for each x, $f(x) \leq g(x) \leq h(x)$.

Show that $\lim_{x \to a} g(x) = L$.

17. Suppose $\lim_{x \to a} f(x) = 0$, and $g(x)$ is bounded. ($\exists\ M > 0$ such that $|\,g(x)\,| < M \ \ \forall x \in \Re$) Show that $\lim_{x \to a} f(x) \cdot g(x) = 0$.

18. State and prove one of the limit theorems for functions.

19. Define what it means for a function f to be continuous at $x = a$. What does it mean for f to be continuous on an interval (a, b)?

20. Properties of continuous functions:

a) +, - , x , / , $\circ$

b) locally bounded

c) 1-1 $\Rightarrow$ monotone

d) Fixed Point Theorem

e) Max - Min Theorem

f) Intermediate Value Theorem

g) characterization of continuity in terms of convergent sequences